Biot bulk coefficient and the micro-inhomogeneity parameter

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Summary

The Biot bulk coefficient (Biot’s $\alpha$) is an important deformation parameter of porous rocks. There are two different ways to measure this coefficient using quasi-static deformation experiments in the laboratory. Only if the rock deforms in a self-similar manner then these two measurements should result in an identical value. However, in the presence of micro-inhomogeneities, one can expect that these two measurements result in different $\alpha$ values. We interpret laboratory measurements where such a difference has been reported and link the difference to the micro-inhomogeneity parameter. The latter is a measure for the deviation from the Biot-Gassmann rock behavior wherein self-similar deformation is assumed.

Introduction

The deformation of a rock is always accomplished by the change of porosity. For micro-inhomogeneous rocks the porosity change is not only dependent on the differential pressure but also on the confining pressure (Sahay, 2013, Eq. 29). In this case the pressure equations that relate the confining ($p^c$) and fluid ($p^f$) pressure to the volumetric deformation of the solid ($\epsilon$) and the increment of fluid content ($\zeta$) is presented in the form (Sahay, 2013, Eqs. 40 and 41)

\begin{align*}
-p^c &= K_0 \epsilon - \alpha^* p^f \\
-\frac{1}{M^*} p^f &= \alpha \epsilon - \zeta,
\end{align*}

where the Biot bulk coefficient $\alpha$ relates the drained rock bulk modulus ($K_0$) to the solid-phase bulk modulus ($K_s$) via $K_0 = (1 - \alpha)K_s$. The lumped parameters

\begin{align*}
\alpha^* &= \eta_0 + n(\alpha - \eta_0) \\
\frac{1}{M^*} &= \frac{\eta_0}{K_T} + \frac{\alpha^* - \eta_0}{K_s}
\end{align*}

have been introduced as in Müller and Sahay (2014). These parameters depend on the micro-inhomogeneity parameter $n$. In the Biot-Gassmann limit $n \rightarrow 1$ and thus $\alpha^* \rightarrow \alpha$ as well as $1/M^* \rightarrow 1/M$ degenerates to the fluid storage coefficient of Biot’s theory. Also, in this limiting case the pressure equations become identical with those reported in Wang (2000, Eqs. 2.24, 2.25).
Theory

The pressure equations imply that $\alpha$ and $\alpha^*$ are defined through the two (hypothetical) experiments (see also the definition of the Biot bulk coefficient given in Wang, 2000):

$$
\alpha = \frac{\partial \zeta}{\partial \epsilon} \bigg|_{p^f=0} \quad \alpha^* = \frac{\partial p^c}{\partial p^f} \bigg|_{\epsilon=0} \quad (5a,b)
$$

Noting that $p^d = p^c - p^f$ the latter differential quotient can transformed into

$$
\alpha^* = 1 - \frac{\partial \epsilon}{\partial p^f} \bigg|_{p^d=0} = 1 - \frac{1}{V_0} \frac{\partial V}{\partial p^f} \bigg|_{p^d=0} = 1 - \frac{K_0}{K_s'} \equiv \alpha' . \quad (6)
$$

Here $\alpha'$ denotes the effective pressure coefficient of the bulk volume (Berryman, 1992). It can be obtained from measurements of the drained frame bulk modulus and the unjacketed bulk modulus $K_s'$.

It is important to note that in micro-inhomogeneous porous media the Biot bulk coefficient and the effective pressure coefficient for the bulk volume can be different, $\alpha \neq \alpha'$. This difference can be understood from Eqs. (5a,b). From the deformation experiment described by Eq. (5a) it becomes clear that a change of the increment of fluid content due to a change in the bulk volume at zero fluid pressure only involves the solid phase (as fluid phase is not pressurized and therefore does not take part in the deformation process). Hence, the solid-phase bulk modulus could be determined from a hydrostatic compression experiment ($p^d = 0$) wherein the change of the solid volume ($V_s$) is monitored

$$
\frac{1}{K_s} \equiv - \frac{1}{V_0} \frac{\partial V}{\partial p^d} \bigg|_{p^d=0} . \quad (7)
$$

Conversely, the experiment described by Eq. (5b) implies that both, the confining and the fluid pressure change and both phases take part in the deformation process. Hence, the unjacketed bulk modulus is obtained from a hydrostatic compression experiment wherein the change of the bulk volume is monitored $\frac{1}{K_s'} \equiv - \frac{1}{V_0} \frac{\partial V}{\partial p^f} \bigg|_{p^d=0}$. Clearly, the outcomes of hydrostatic compression tests will depend on which volume change is monitored. The only exception is a self-similar deformation which would imply $\Delta V/V_0 = \Delta V_s/V_{s0}$ and thus $K_s = K_s'$. This is precisely the assumption Gassmann (1951) makes in the development of his theory (his Eq. 45).

Without specifying any micro-geometry, we note that a self-similar deformation requires that the deformational potential energy is uniform within the deformed rock sample (as otherwise subvolumes with higher deformational potential energy would experience an extra strain). Any localization of the deformational potential energy implies a non-self-similar deformation and hence a difference in $\alpha$ and $\alpha'$. We can envisage many micro-inhomogeneity scenarios which lead to a violation of the self-similarity assumption. For example, as micro-inhomogeneities can be present in the form of stiffness fluctuations in the pore space of the rock (say, air bubbles dispersed at pore-scale) $K_s'$ is expected to depend on the fluid bulk modulus $K_f$, while $K_s$ cannot depend on $K_f$. A measure for the degree of micro-inhomogeneity is the micro-inhomogeneity parameter $n$. From the above discussion we expect that the difference between $\alpha$ and $\alpha'$ is proportional to the degree of micro-inhomogeneity and thus $n$. Substituting Eq. (6) in Eq. (3) and solving for $n$ yields to (see also Müller and Sahay, 2014, Eq. 12)

$$
n = 1 - \frac{\alpha - \alpha'}{\alpha - \eta_0} . \quad (8)
$$

In the reminder of this paper we are going to analyze experimental data where a difference between $\alpha$ and $\alpha'$ has been found.
Modeling laboratory data

Blöcher et al. (2014) measure a comprehensive set of poroelastic parameters for Bentheimer sandstone. Their direct measurement of the Biot bulk coefficient corresponds to $\alpha$ and their indirect measurement is $\alpha'$. Most notably, $\alpha$ and $\alpha'$ differ over the whole range of applied differential pressure. This difference indicates that this rock is not conform with the Biot-Gassmann theory wherein $\alpha = \alpha'$ is assumed. Blöcher et al. (2014) relate the cause of this difference to the presence of micro-inhomogeneities. Using Eq. (8) the micro-inhomogeneity parameter can be directly evaluated (Figure 1).

![Figure 1: Measurements for $\alpha$ and $\alpha'$ for Bentheimer sandstone (taken from Blöcher et al. (2014). Also shown is the micro-inhomogeneity parameter estimation based on Eq. (8).](image)

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References


